MATHEMATICS
ESSAY PROBLEMS

Name : _____ **Index Number :** _____

Country : _____



19th International Mathematics and Science Olympiad
Indonesia
30 November 2022

Instructions:

1. Write your name, country and index number on every page of this booklet.
2. Write your detailed solution in Arabic Numerical or English in the space provided for each question in this booklet. If you need more space for your working solutions, you may use the reverse side of each page, but please write "next side continued" in the last line.
3. There are 13 questions in this paper.
4. Each question is worth 3 marks and partial credits may be awarded.
5. Diagrams are NOT drawn to scale. They are intended only as aids.
6. You have 90 minutes to complete this paper.
7. Use black pen or blue pen or pencil to write your answer.

Do not turn over this page until you are told to do so.

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1. A bakery sells bread for $5 a piece in the morning, but sells them at a 20% discount during the afternoon. The owner of the bakery noticed that the amount of breads sold in the afternoon increases by 350% compared to the morning and that the total revenue for the whole day is $4646. How many breads does the bakery sell for the whole day?

Answer: _____ breads

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2. John has a 24-hour digital clock which shows the time from 00:00 to 23:59 each day. How many times in a day is the sum of the digits of the current hour equal to the sum of the digits of the current minute?

(An example is 12:30, where the sum of the digits of the current hour: $1 + 2 = 3$ is equal to the sum of the digits of the current minute: $3 + 0 = 3$.)

Answer: _____ times

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3. Let N be the largest 2022-digit number that is a multiple of 81. What are the last two digits of N?

Answer: _____

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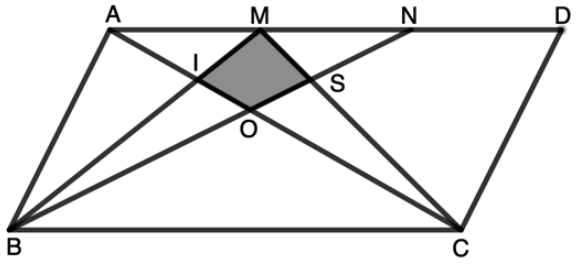
4. How many ways can we put exactly one "+" sign and zero, one or several "×" signs inside $\square$ of the following expression: $1\square 2\square 1\square 2\square 1$, such that the result is an odd number?

Answer: _____ ways

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5. In the figure below, $ABCD$ is a parallelogram that has an area of 600 cm^2. The points M and N are on the line $\overline{AD}$ such that $AM = MN = ND$, the point O is the intersection of $\overline{AC}$ and $\overline{BN}$, the point I is the intersection of $\overline{AC}$ and $\overline{BM}$ and the point S is the intersection of $\overline{BN}$ and $\overline{CM}$. What is the area, in cm^2, of quadrilateral $IMSO$?



Answer: _____ cm^2

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6. There are three problems in a mathematics contest. Students win gold, silver or bronze medals if they are able to solve 3, 2 or 1 problem(s) respectively. If each problem is solved by exactly 80 students and there are exactly 144 medalists, what is the positive difference between the number of gold and bronze medalists?

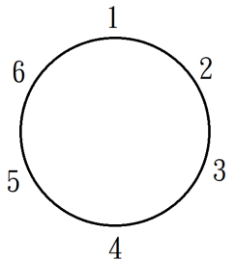
Answer: _____

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7. Write the numbers 1, 2, 3, 4, 5, and 6 around a circle in clockwise direction. We always start by removing the number 1 and continue to do this process of removing the second remaining number, also in clockwise direction, until the last number remains.

For example,



Turn 1: Remove Number 1

Turn 2: Keep 2, Remove 3

Turn 3: Keep 4, Remove 5

Turn 4: Keep 6, Remove 2

Turn 5: Keep 4, Remove 6

So, the last remaining number is 4.

If the numbers from 1 to 2022 are written on a circle in clockwise direction and the same process is applied, what will be the last number that remains?

Answer: _____

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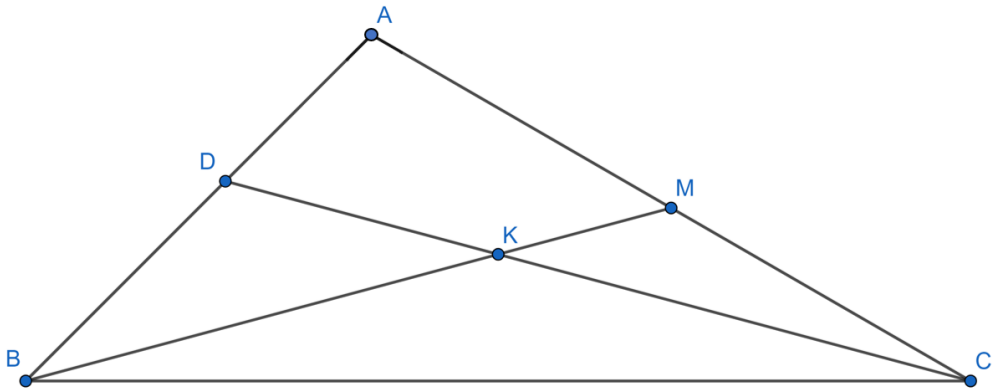
8. There are 25 oranges, having different sizes, which are placed on a 5 by 5 grid. First, we take the biggest orange from each of the rows. Thus, we now have 5 oranges, and then among them, we locate the smallest one among these 5 oranges and mark it as X. After which, we place these 5 oranges back into their original positions. Similarly, take the smallest orange from each of the columns. After doing this, we again have 5 oranges, and then among them, we locate the biggest one among these 5 oranges and mark it as Y. After which, we again place these 5 oranges back into their original positions. Is it possible to determine which orange is bigger, X or Y? Justify your answer.

Answer: _____

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9. Given a triangle ABC with $105^o < \angle A < 115^o$. The median BM intersects the bisector CD at K.
If $KB = KC$, list all possible value of the measure, in degrees, of $\angle ABC + \angle ACB$?



Answer: _____ o

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10. How many ways are there to select four distinct integers a, b, c and d from $1, 2, \ldots, 600$ such that a, b, c and d forms a strictly increasing geometric sequence whose common ratio is positive integer?

Note: Two examples of a geometric sequence are 1, 2, 4, 8, 16, … having a common ratio of 2 and 2, 6, 18, 54, 162, … having a common ratio of 3.

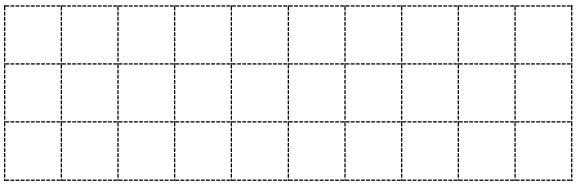
Answer: _____ ways

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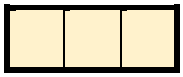
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11. How many different arrangements are there to cover the 3 × 10 rectangle using ten 3 × 1 rectangles without overlapping?

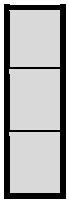
Note: Each 3 × 1 rectangle can be placed horizontally or vertically.



3×10 rectangle



3×1 rectangle placed horizontally



3×1 rectangle placed vertically

Answer: _____ arrangements

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12. A number is called "$IMSO_2$" if it can be written as the sum of 2 consecutive positive integers.

A number is called "$IMSO_3$" if it can be written as the sum of 3 consecutive positive integers.

A number is called "$IMSO_4$" if it can be written as the sum of 4 consecutive positive integers.

A number is called "$IMSO_5$" if it can be written as the sum of 5 consecutive positive integers.

For example, the number 15 is an $IMSO_2$ because it can be written as $15 = 7 + 8$.

Moreover, 15 is also an $IMSO_3$ because it can be written as $15 = 4 + 5 + 6$.

Furthermore, 15 is also an $IMSO_5$ because it can be written as $15 = 1 + 2 + 3 + 4 + 5$.

However, 15 is not an $IMSO_4$ because it cannot be written as the sum of 4 consecutive positive integers.

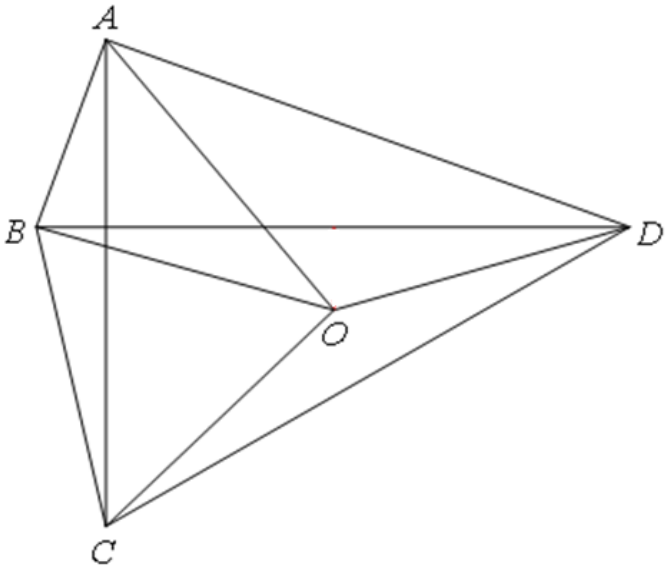
From 1 to 2022, how many of them are $IMSO_2$, $IMSO_3$ and $IMSO_5$ but not $IMSO_4$ at the same time?

Answer: _____

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13. Let O be a point inside a convex quadrilateral $ABCD$ with perpendicular diagonals such that $OB = OD$. If the area of the triangle ABC is 45 cm^2, the area of the triangle AOD is 120 cm^2, and the area of the triangle OCD is 60 cm^2, then what is the area, in cm^2, of the triangle AOC?



Answer: _____ cm^2